

Tennessee Higher Education Commission

Supplementary Task: Applications of Systems of Linear Equations with Infinitely Many Solutions

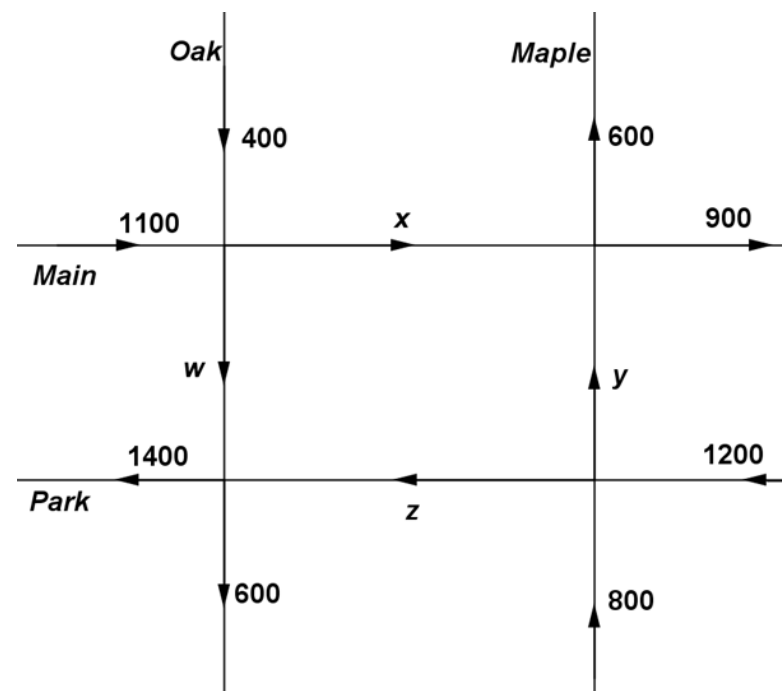
Students often fail to understand the significance of “infinitely many solutions” for a system of linear equations. In most cases, students may be able to recognize that such a system has infinitely many solutions, but do not know how to describe those solutions or why anyone would want to know a general form for those solutions. This task is designed to provide a context for a system of linear equations in which the “infinitely many solutions” case is vital to the context of the problem.

This examples supports the objective:

Use systems of two or more equations or inequalities to solve problems using tables, graphs, and algebraic properties. Interpret intersections/regions in the context of the problem in the Core to College Master Syllabus for Core-Aligned College Algebra.

Example: Traffic Flow

The figure below shows the flow of traffic in a city during the rush hours on a typical weekday. The arrows indicate the direction of the flow of traffic on each one-way road. The average number of vehicles per hour entering and leaving each intersection is indicated beside each road. Oak and Maple streets can each handle up to 1000 cars per hour without congestion. Main and Park streets can each handle up to 1600 cars per hour without congestion. Traffic lights installed at each of the four intersections control the flow of traffic.



- Explicitly state a general expression involving the rates of flow (x , y , z , and w) and suggest two different possible flow patterns that will ensure no traffic congestion.
- Suppose there is a water main break on Oak Street between Main and Park, and the traffic along Oak is narrowed to one lane. By narrowing to one lane, the maximum number of cars per hours on Oak Street is reduced to 600. Find two different possible flow patterns that will ensure no traffic congestion with this restriction.

Prior Knowledge Needed:

Students will need to know at least one method for solving a system of linear equations. Students will need some background knowledge regarding solving systems of linear equations with infinitely many solutions.

Students may need assistance understanding the vocabulary associated with traffic flow patterns.

Prerequisite Common Core State Standards for Mathematical Content that support this example

Create equations that describe numbers or relationships

Solve systems of equations

*The complete Common Core State Standards for High School Mathematics can be found in <http://www.corestandards.org/Math/>.

Solution

Setting up the equations:

Each intersection will give rise to a linear equation. The number of cars flowing in to an intersection must equal the number of cars flowing out of the intersection. We will call the intersection of Oak and Main intersection "A", then move in a counterclockwise direction to name intersections B, C, and D.

Intersection	Flow Into Intersection	Flow Out of Intersection
A	$1100 + 400$	$x + w$
B	$w + z$	$1400 + 600$
C	$800 + 1200$	$y + z$
D	$x + y$	$600 + 900$

Thus, our system of equations is:

$$x + w = 1500$$

$$w + z = 2000$$

$$y + z = 2000$$

$$x + y = 1500$$

Solving the system:

Students may use the methods of elimination or substitution to solve the system by hand. If students are familiar with matrices, they may put the augmented matrix associated with the system in reduced row echelon form either by hand or using technology.

This system will reduce to an equivalent system of three equations in four variables, so we must choose an arbitrary value for one of the variables. If we let w be the arbitrary variable, then the solution to our system is:

$$x = 1500 - w$$

$$y = w$$

$$z = 2000 - w$$

Limitations on the values of the variables:

To write a solution to the problem, we must consider whether there are any constraints on the variables. In the problem, we are told that Oak and Maple streets can each handle up to 1000 cars, and Main and Park streets can each handle up to 1600 cars. Thus, in our solutions, we need to take these constraints into account.

The variable w describes the flow of traffic on Oak Street, so we know that our values of w cannot exceed 1000. Similarly, neither x nor z can exceed 1600, and y cannot exceed 1000. Since all four variables represent the number of cars using the street, we know that all four variables must be non-negative.

To describe the flow patterns, we will need to choose a value for w and then calculate the values of x , y , and z based on the value chosen for w . Note that since $z = 2000 - w$ and since z cannot exceed 1600, we know that w must be at least 400 in order to determine a valid value for z .

This will also have consequences for the possible values of x and y .

Solution to part (a):

The general expression for the rates of flow are given as:

$$x = 1500 - w$$

$$y = w$$

$$z = 2000 - w$$

where $400 \leq w \leq 1000$.

Flow patterns will depend on the number chosen for w . For example, if a student chooses $w = 500$, then we know that $x = 1000$, $y = 500$, and $z = 1500$.

Solution to part (b):

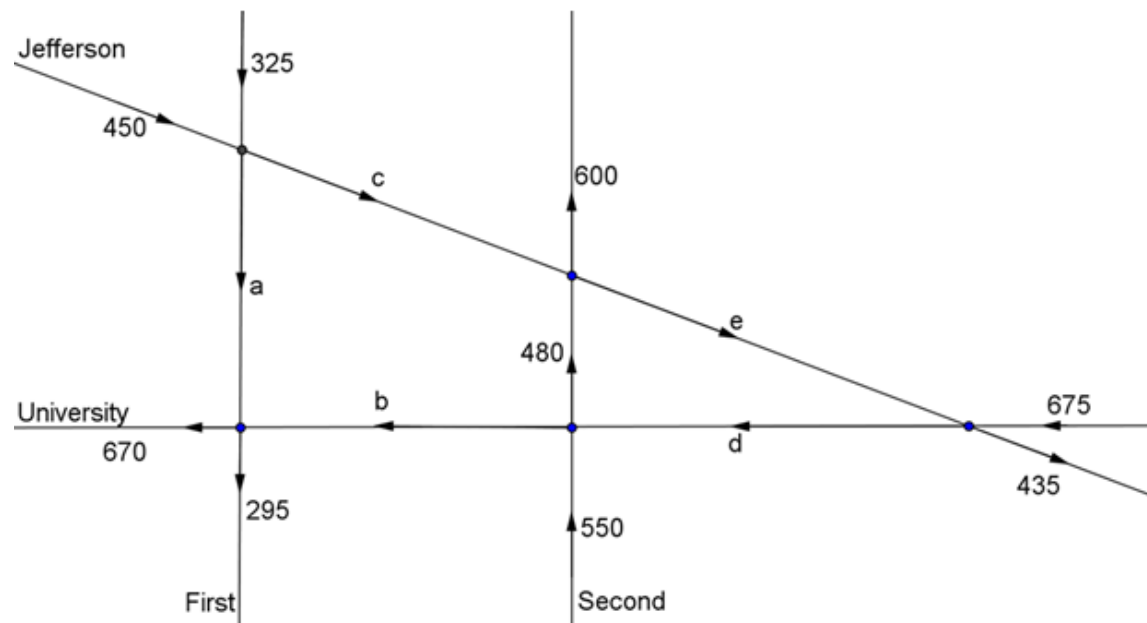
Reducing the maximum number of cars on Oak Street between Main and Park to 600 cars per hour will affect our possible solutions, but will not affect the set-up of the system or the general solution.

Since the maximum number of cars on Oak Street is now 600, our possible values for w are now limited to $400 \leq w \leq 600$. Thus, a sample flow pattern using, say, $w = 450$ would be:

$$w = 450, x = 1150, y = 450, \text{ and } z = 1550.$$

Task for Student Work

The figure below shows the flow of traffic in a city during the rush hours on a typical weekday. The arrows indicate the direction of the flow of traffic on each one-way road. The average number of vehicles per hour entering and leaving each intersection is indicated beside each road. Jefferson and University streets can each handle up to 800 cars per hour without congestion. First and Second streets can each handle up to 750 cars per hour without congestion. Traffic lights installed at each of the five intersections control the flow of traffic.



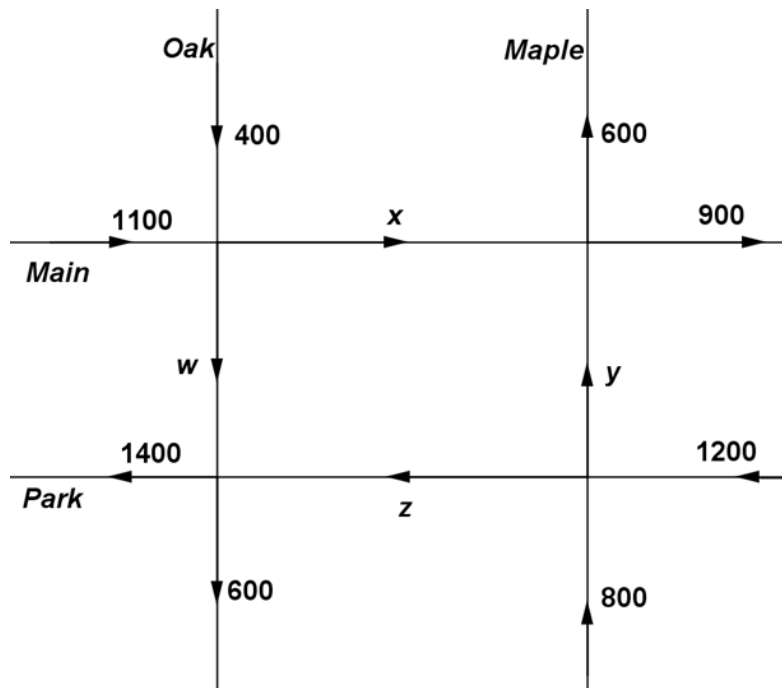
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Applications of Linear Equations

Example: Traffic Flow

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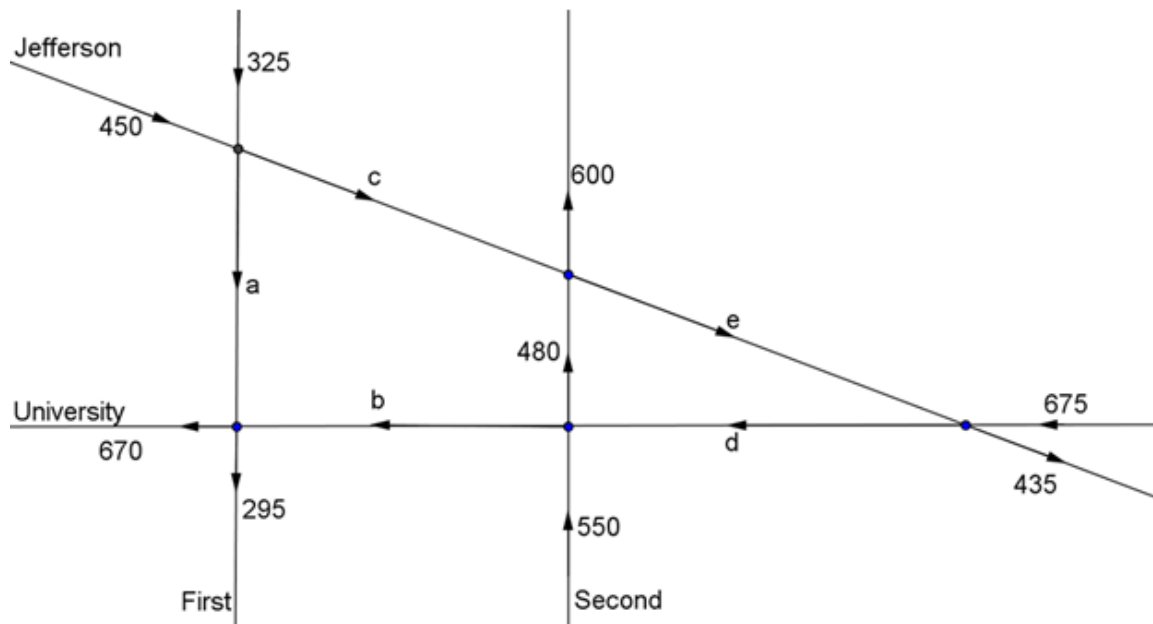
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Applications of Linear Equations

Student Task

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